I. PROBLEM SET: II (TOTAL MARKS: 20)

Due on 8th February 2019

Q1. Consider the pure state

$$\left|\psi^{AB}\right\rangle = \tfrac{1+\sqrt{6}}{2\sqrt{6}}\left|00\right\rangle + \tfrac{1-\sqrt{6}}{2\sqrt{6}}\left|01\right\rangle + \tfrac{\sqrt{2}-\sqrt{3}}{2\sqrt{6}}\left|10\right\rangle + \tfrac{\sqrt{2}+\sqrt{3}}{2\sqrt{6}}\left|11\right\rangle$$

where the first qubit belongs to Alice and second to Bob.

1. Calculate $\rho^{AB} \rightarrow$ verify the purity of this state. [3]
2. Compute $\rho_A = Tr_B(|\psi\rangle\langle\psi|)$ & $\rho_B = Tr_A(|\psi\rangle\langle\psi|)$ [2]
3. Find the Schmidt decomposition of $|\psi\rangle$ [2]

Q2. Consider the state $|\psi\rangle = \frac{1}{\sqrt{3}}\left(|00\rangle + |01\rangle + |10\rangle\right)$. Do the Schmidt decomposition of this state using the Singular Value Decomposition as taught in class. [5]

Q3. Consider the three qubit GHZ state

$$|\psi\rangle = \tfrac{1}{\sqrt{2}}\left[|\uparrow\rangle_1 |\uparrow\rangle_2 |\uparrow\rangle_3 + |\downarrow\rangle_1 |\downarrow\rangle_2 |\downarrow\rangle_3\right] = \tfrac{1}{\sqrt{2}}\left[|000\rangle + |111\rangle\right]$$

1. Find out the reduced density matrix $\rho_{AB} = Tr_C(\rho_{ABC})$ [2]
2. Calculate the Von-Neumann entropy , S(ρ_{AB}) [2]
3. Find out $\rho_A = Tr_{BC}(\rho_{ABC})$. Calculate S$(\rho_A)$ [2]
4. Comment on the nature of entanglement of the GHZ state. [2]